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Graduate Texts in Mathematics

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J.L. Alperin
with Rowen B. Bell

Groups and Representations

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Preface

This book is based on a first-year graduate course given regularly by the first author at the University of Chicago, most recently in the autumn quarters of 1991, 1992, and 1993. The lectures given in this course were expanded and prepared for publication by the second author.

The aim of this book is to provide a concise yet thorough treatment of some topics from group theory and representation theory with which every mathematician should be well acquainted. Of course, the topics covered naturally reflect the viewpoints and interests of the authors; for instance, we make no mention of free groups, and the emphasis throughout is admittedly on finite groups. Our hope is that this book will enable graduate students from every mathematical field, as well as bright undergraduates with an interest in algebra, to solidify their knowledge of group theory.

As the course on which this book is based is required for all incoming mathematics graduate students at Chicago, we make very modest assumptions about the algebraic background of the reader. A nodding familiarity with groups, rings, and fields, along with some exposure to elementary number theory and a solid knowledge of linear algebra (including, at times, familiarity with canonical forms of matrices), should be sufficient preparation.

We now give a brief summary of the book's contents. The first four chapters are devoted to group theory. Chapter 1 contains a review (largely without proofs) of the basics of group theory, along with material on automorphism groups, semidirect products, and group actions. These latter concepts are among our primary tools in the book and are often not covered adequately during one's first exposure to group theory. Chapter 2 discusses the structure of the general linear groups and culminates with a proof of the simplicity of the projective special linear groups. An understanding of this material is an essential (but often overlooked) component of any substantive study of group theory; for, as the first author once wrote:

> The typical example of a finite group is $\mathrm{GL}(n, q)$, the general linear group of n dimensions over the field with q elements. The student who is introduced to the subject with other examples is being completely misled. [3, p. 121]

Chapter 3 concentrates on the examination of finite groups through their p-subgroups, beginning with Sylow's theorem and moving on to such results as the Schur-Zassenhaus theorem. Chapter 4 starts with the Jordan-Hölder theorem and continues with a discussion of solvable and nilpotent groups. The final two chapters focus on finite-dimensional algebras and the representation theory of finite groups. Chapter 5 is centered around Maschke's theorem and Wedderburn's structure theorems for semisimple algebras. Chapter 6 develops the ordinary character theory of finite groups, including induced characters, while the Appendix treats some additional topics in character theory that require a somewhat greater algebraic background than does the core of the book.

We have included close to 200 exercises, and they form an integral part of the book. We have divided these problems into "exercises" and "further exercises;" the latter category is generally reserved for exercises that introduce and develop theoretical concepts not included in the text. The level of the problems varies from routine to difficult, and there are a few that we do not expect any student to be able to handle. We give no indication of the degree of difficulty of each exercise, for in mathematical research one does not know in advance what amount of work will be required to complete any step! In an effort to keep our exposition self-contained, we have strived to keep references in the text to the exercises at a minimum.

The sections of this book are numbered continuously, so that Section 4 is actually the first section of Chapter 2, and so forth. A citation of the form "Proposition Y" refers to the result of that name in the current section, while a citation of the form "Proposition X.Y" refers to Proposition Y of Section X.

We would like to extend our thanks to: Michael Maltenfort and Colin Rust, for their thought-provoking proofreading and their many constructive suggestions during the preparation of this book; the students in the first author's 1993 course, for their input on an earlier draft of this book which was used as that course's text; Efim Zelmanov and the students in his 1994 Chicago course, for the same reason; and the University of Chicago mathematics department, for continuing to provide summer support for graduate students, as without such support this book would not have been written in its present form. We invite you to send notice of errors, typographical or otherwise, to the second author at `bell@math.uchicago.edu`.

In remembrance of a life characterized by integrity, devotion to family, and service to community, the second author would like to dedicate this book to David Wellman (1953–1995).

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