

Oscar Zariski

# Algebraic Surfaces

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Oscar Zariski

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With Appendices by  
S. S. Abhyankar, J. Lipman, and D. Mumford



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