Oscar Zariski

Algebraic Surfaces

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Oscar Zariski

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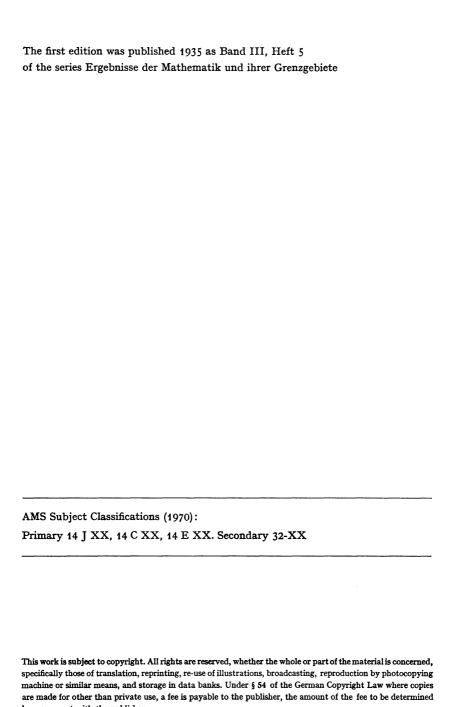
Algebraic Surfaces

Second Supplemented Edition

With Appendices by S. S. Abhyankar, J. Lipman, and D. Mumford



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