

MEMOIRS

of the
American Mathematical Society

Number 783

Dynamics of Topologically Generic Homeomorphisms

Ethan Akin
Mike Hurley
Judy A. Kennedy



July 2003 • Volume 164 • Number 783 (end of volume) • ISSN 0065-9266

American Mathematical Society
Providence, Rhode Island

Contents

Introduction	1
0.1. Overview	1
0.2. Description of results	2
0.3. Brief remarks about techniques	5
0.4. Comparison with the smooth case	6
0.5. Standing notation	7
0.6. Road map	8
0.7. Comments	8
Chapter 1. Attractors and Chain Recurrence	11
1.1. Chain recurrence	11
1.2. Attractors	13
1.3. Attractor-repellor pairs	15
1.4. Initial and terminal chain components	16
1.5. The space(s) of chain components	17
1.6. Summary	18
Chapter 2. Periodic Decompositions and Adding Machines	21
2.1. Periodic decompositions	21
2.2. The sets Π_∞ and $\Pi_{\infty, \infty}$	22
2.3. Adding machines	23
2.4. Decompositions of \mathcal{U} type	25
2.5. Periodicity conditions	28
Chapter 3. Semicontinuity and Homogeneity	31
3.1. Semicontinuity	31
3.2. Prolongation	32
3.3. The automorphism group	33
3.4. Homogeneity conditions	35
3.5. Showing $\Omega(f) = \mathcal{C}(f)$	36
Chapter 4. Crushing Arguments	39
4.1. Sponges	39
4.2. Crushing	44
Chapter 5. Topological Horseshoes	53
5.1. Connected successions of subsets	53
5.2. Topological horseshoes	54
5.3. Perturbing to a horseshoe	57
Chapter 6. Generic Homeomorphisms	65

6.1.	The classes \mathcal{H}_1 and $\mathcal{H}_{1,s}$	65
6.2.	The class $\mathcal{H}_{3,s}$	70
6.3.	Dynamic isolation	73
6.4.	Attractor boundaries are quasi-attractors	73
6.5.	Shift extensions and the class \mathcal{H}_4	75
Chapter 7.	Almost Equicontinuity	79
7.1.	Chain continuity	81
Chapter 8.	Cantor Sets	85
8.1.	Aperiodicity and the class \mathcal{H}_5	85
8.2.	The class \mathcal{H}_6	86
8.3.	Rohlin property	89
8.4.	The class $\mathcal{H}_{3,c}$	90
Chapter 9.	The Circle	97
9.1.	Background	97
9.2.	The class \mathcal{H}_1 on S^1	99
9.3.	Relative Rohlin property	103
Chapter 10.	Crushing the Chain Recurrent Set	107
Chapter 11.	Generic Homeomorphisms on Manifolds	115
11.1.	The class \mathcal{H}_8	115
11.2.	The class \mathcal{H}_{man}	116
11.3.	Anosov homeomorphisms	116
Chapter 12.	Generic Mappings on Manifolds	121
Bibliography		125
Index		129