

THEORY OF LINEAR OPERATORS IN HILBERT SPACE

N. I. Akhiezer and
I. M. Glazman

Translated from the Russian by
MERLYND NESTELL

TWO VOLUMES BOUND AS ONE

DOVER PUBLICATIONS, INC.
NEW YORK

CONTENTS

Chapter I. HILBERT SPACE

1. Linear Spaces	1
2. The Scalar Product	2
3. Some Topological Concepts	4
4. Hilbert Space	5
5. Linear Manifolds and Subspaces	7
6. The Distance from a Point to a Subspace	8
7. Projection of a Vector on a Subspace	10
8. Orthogonalization of a Sequence of Vectors	13
9. Complete Orthonormal Systems	17
10. The Space L^2	21
11. Complete Orthonormal Systems in L^2	24
12. The Space L^2_σ	27
13. The Space of Almost Periodic Functions	29

Chapter II. LINEAR FUNCTIONALS AND BOUNDED LINEAR OPERATORS

30

14. Point Functions	30
15. Linear Functionals	32
16. The Theorem of F. Riesz	33
17. A Criterion for the Closure in H of a Given System of Vectors	35
18. A Lemma Concerning Convex Functionals	36
19. Bounded Linear Operators	39
20. Bilinear Functionals	40
21. The General Form of a Bilinear Functional	42
22. Adjoint Operators	43
23. Weak Convergence in H	44
24. Weak Compactness	46

Chapter II (continued)

25. A Criterion for the Boundedness of an Operator	48
26. Linear Operators in a Separable Space	48
27. Completely Continuous Operators	56
28. A Criterion for Complete Continuity of an Operator	57
29. Sequences of Bounded Linear Operators	61

*Chapter III. PROJECTION OPERATORS AND
UNITARY OPERATORS*

63

30. Definition of a Projection Operator	63
31. Properties of Projection Operators	63
32. Operations Involving Projection Operators	65
33. Monotone Sequences of Projection Operators	68
34. The Aperture of Two Linear Manifolds	69
35. Unitary Operators	72
36. Isometric Operators	73
37. The Fourier-Plancherel Operator	74

*Chapter IV. GENERAL CONCEPTS AND PROPOSITIONS
IN THE THEORY OF LINEAR OPERATORS*

78

38. Closed Operators	78
39. The General Definition of an Adjoint Operator	79
40. Eigenvectors, Invariant Subspaces and Reducibility of Linear Operators	81
41. Symmetric Operators	85
42. More about Isometric and Unitary Operators	87
43. The Concept of the Spectrum (Particularly of a Self-Adjoint Operator)	88
44. The Resolvent	91
45. Conjugation Operators	94
46. The Graph of an Operator	95
47. Matrix Representations of Unbounded Symmetric Operators	98

Chapter IV (continued)

- | | |
|---|-----|
| 48. The Operation of Multiplication by the Independent Variable | 103 |
| 49. A Differential Operator | 106 |
| 50. The Inversion of Singular Integrals | 114 |

Chapter V. SPECTRAL ANALYSIS OF COMPLETELY CONTINUOUS OPERATORS

117

- | | |
|--|-----|
| 51. A Lemma | 117 |
| 52. Properties of the Eigenvalues of Completely Continuous Operators in \mathbb{R} | 118 |
| 53. Further Properties of Completely Continuous Operators | 122 |
| 54. The Existence Theorem for Eigenvectors of Completely Self-Adjoint Operators | 124 |
| 55. The Spectrum of a Completely Continuous Self-Adjoint Operator in \mathbb{R} | 127 |
| 56. Completely Continuous Normal Operators | 129 |
| 57. Applications to the Theory of Almost Periodic Functions | 132 |

BIBLIOGRAPHY

139

INDEX

145