IN MEMORIAM

R.H.F. and R.H.K.

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A General Topology Workbook

Springer Science+Business Media, LLC

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Library of Congress Cataloging-in-Publication Data

Adamson, Iain T.
A general topology workbook / Iain T. Adamson
p. cm.
ISBN 978-0-8176-3844-3 ISBN 978-0-8176-8126-5 (eBook)
DOI 10.1007/978-0-8176-8126-5
acid-free
1. Topology. I. Title.
QA611.A346 1995
514'.322–dc20 95-33319
CIP

Printed on acid-free paper
©1996 Springer Science+Business Media New York
Originally published by Birkhäuser Boston in 1996

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ISBN 978-0-8176-3844-3 SPIN 10782858

Typeset by the author in LaTeX.

9 8 7 6 5 4 3 2

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INTRODUCTION

This book has been called a *Workbook* to make it clear from the start that it is not a conventional textbook. Conventional textbooks proceed by giving in each section or chapter first the definitions of the terms to be used, the concepts they are to work with, then some theorems involving these terms (complete with proofs) and finally some examples and exercises to test the readers' understanding of the definitions and the theorems. Readers of this book will indeed find all the conventional constituents—definitions, theorems, proofs, examples and exercises—but not in the conventional arrangement.

In the first part of the book will be found a quick review of the basic definitions of general topology interspersed with a large number of exercises, some of which are also described as theorems. (The use of the word *Theorem* is not intended as an indication of difficulty but of importance and usefulness.) The exercises are deliberately not "graded"—after all the problems we meet in mathematical "real life" do not come in order of difficulty; some of them are very simple illustrative examples; others are in the nature of tutorial problems for a conventional course, while others are quite difficult results. No solutions of the exercises, no proofs of the theorems are included in the first part of the book—this is a *Workbook* and readers are invited to try their hand at solving the problems and proving the theorems for themselves. I have been persuaded, with some reluctance, to offer suggestions about how to tackle the exercises which are not entirely straightforward; really dedicated *Workbook*-ers should ignore these! The second part of the book contains complete solutions to all but the most utterly trivial exercises and complete proofs of the theorems.

It has been widely recognised that general topology is a branch of mathematics particularly well adapted to independent study based on material carefully prepared by a teacher who otherwise gives minimal assistance. The most celebrated practitioner of this method was R.L. Moore (1882-1974) whose success with it is legendary. One factor

in Moore's success must have been his insistence on hand-picking his students, preferring those who came to him *tabula rasa.* Few of his imitators and followers have enjoyed the luxury of being able to do this and have had to adapt his method to their more mundane circumstances. This book has grown from my attempts to provide a self-learning introduction to general topology for several generations of students in The University of Dundee, not all of whom would have been selected by Moore—though all of them responded enthusiastically to the method.

My Dundee students were presented, by instalments, with the introductory material and the exercises (though with much less generous hints than appear in the book) and were given the solutions only after the class had met to discuss the problems. Not all the students solved every problem for themselves, but there was a much higher participation rate than in conventional lecture courses. Two students who tackled the course by virtually unsupervised reading had particularly marked success.

In a class situation it is easy to ensure that students do not see the solutions to the exercises before they have tried to solve them; it is not so easy when exercises and solutions appear between the same two covers. But my readers are reminded that this is a *Workbook* and they are warmly invited to work at the exercises before turning to the solutions.

I must put on record my thanks to the Dundee students who have worked so cheerfully with the contents of this book, especially Malcolm Dobson and Ross Anderson. I am also deeply indebted to my friend Keith Edwards for his meticulous assistance, both typographical and mathematical. My gratitude to my wife for her encouragement during the writing and typesetting of this book is beyond measure.

Dundee, Scotland
August 1995

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