

CONTENTS

Preface	ix
List of Spaces and Norms	xii
1. PRELIMINARIES	1
Notation	1
Topological Vector Spaces	3
Normed Spaces	4
Spaces of Continuous Functions	10
The Lebesgue Measure in \mathbb{R}^n	13
The Lebesgue Integral	16
Distributions and Weak Derivatives	19
2. THE LEBESGUE SPACES $L^p(\Omega)$	23
Definition and Basic Properties	23
Completeness of $L^p(\Omega)$	29
Approximation by Continuous Functions	31
Convolutions and Young's Theorem	32
Mollifiers and Approximation by Smooth Functions	36
Precompact Sets in $L^p(\Omega)$	38
Uniform Convexity	41
The Normed Dual of $L^p(\Omega)$	45
Mixed-Norm L^p Spaces	49
The Marcinkiewicz Interpolation Theorem	52

3. THE SOBOLEV SPACES $W^{m,p}(\Omega)$	59
Definitions and Basic Properties	59
Duality and the Spaces $W^{-m,p'}(\Omega)$	62
Approximation by Smooth Functions on Ω	65
Approximation by Smooth Functions on \mathbb{R}^p	67
Approximation by Functions in $C_0^\infty(\Omega)$	70
Coordinate Transformations	77
4. THE SOBOLEV IMBEDDING THEOREM	79
Geometric Properties of Domains	81
Imbeddings by Potential Arguments	87
Imbeddings by Averaging	93
Imbeddings into Lipschitz Spaces	99
Sobolev's Inequality	101
Variations of Sobolev's Inequality	104
$W^{m,p}(\Omega)$ as a Banach Algebra	106
Optimality of the Imbedding Theorem	108
Nonimbedding Theorems for Irregular Domains	111
Imbedding Theorems for Domains with Cusps	115
Imbedding Inequalities Involving Weighted Norms	119
Proofs of Theorems 4.51–4.53	131
5. INTERPOLATION, EXTENSION, AND APPROXIMATION THEOREMS	135
Interpolation on Order of Smoothness	135
Interpolation on Degree of Sumability	139
Interpolation Involving Compact Subdomains	143
Extension Theorems	146
An Approximation Theorem	159
Boundary Traces	163
6. COMPACT IMBEDDINGS OF SOBOLEV SPACES	167
The Rellich-Kondrachov Theorem	167
Two Counterexamples	173
Unbounded Domains — Compact Imbeddings of $W_0^{m,p}(\Omega)$	175
An Equivalent Norm for $W_0^{m,p}(\Omega)$	183
Unbounded Domains — Decay at Infinity	186
Unbounded Domains — Compact Imbeddings of $W^{m,p}(\Omega)$	195
Hilbert-Schmidt Imbeddings	200

7. FRACTIONAL ORDER SPACES	205
Introduction	205
The Bochner Integral	206
Intermediate Spaces and Interpolation — The Real Method	208
The Lorentz Spaces	221
Besov Spaces	228
Generalized Spaces of Hölder Continuous Functions	232
Characterization of Traces	234
Direct Characterizations of Besov Spaces	241
Other Scales of Intermediate Spaces	247
Wavelet Characterizations	256
8. ORLICZ SPACES AND ORLICZ-SOBOLEV SPACES	261
Introduction	261
N-Functions	262
Orlicz Spaces	266
Duality in Orlicz Spaces	272
Separability and Compactness Theorems	274
A Limiting Case of the Sobolev Imbedding Theorem	277
Orlicz-Sobolev Spaces	281
Imbedding Theorems for Orlicz-Sobolev Spaces	282
References	295
Index	301