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Manifolds, Tensor Analysis, and Applications

Second Edition

Springer

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Contents

Background Notation

The reader is assumed to be familiar with the usual notations of set theory such as $\in$, $\subset$, $\cup$, $\cap$ and with the concept of a mapping. If A and B are sets and if f: A→B is a mapping, we write $a \mapsto f(a)$ for the effect of the mapping on the element of $a \in A$; "iff" stands for "if and only if" (= "if" in definitions). Other notations we shall use without explanation include the following:

◆	end of an example or remark
■	end of a proof
▼	proof of a lemma is done, but the proof of the theorem goes on
$\mathbb{R}$, $\mathbb{C}$	real, complex numbers
$\mathbb{Z}$, $\mathbb{Q}$	integers, rational numbers
$A \times B$	Cartesian product
$\mathbb{R}^n$, $\mathbb{C}^n$	Euclidean n-space, complex n-space
$(x^1, \ldots, x^n) \in \mathbb{R}^n$	point in $\mathbb{R}^n$
$A \subset B$	set theoretic containment (means same as $A \subseteq B$)
$A \setminus B$	set theoretic difference
I or Id	identity map
$f^{-1}(B)$	inverse image of B under f
$\Gamma_f = \{(x,f(x)) \mid x \in \text{domain of } f\}$	graph of f
inf A	infinimum (greatest lower bound) of the set $A \subset \mathbb{R}$
sup A	supremum (least upper bound) of $A \subset \mathbb{R}$
$e_1, \ldots, e_n$	basis of an n-dimensional vector space
ker T, range T	kernel and range of a linear transformation T
$D_r(m)$	open ball about m of radius r
$B_r(m)$	closed ball of radius r (also denoted $\overline{D}_r(m)$).