Richard A. Holmgren

A First Course in Discrete Dynamical Systems

Second Edition

With 56 Figures

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