

Richard A. Holmgren

A First Course in Discrete Dynamical Systems

Second Edition

With 56 Figures



Springer

Richard A. Holmgren
Department of Mathematics
Allegheny College
Meadville, PA 16335-3902
USA
rholmgre@allegheny.edu

Editorial Board
(North America):

S. Axler
Mathematics Department
San Francisco State University
San Francisco, CA 94132
USA

F.W. Gehring
Mathematics Department
East Hall
University of Michigan
Ann Arbor, MI 48109-1109
USA

K.A. Ribet
Mathematics Department
University of California at Berkeley
Berkeley, CA 94720-3840
USA

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Contents

Preface	vii
Rationale for the new edition	vii
How to use this book	viii
Acknowledgments	ix
List of Symbols	xi
1. Introduction	1
1.1. Phase Portraits	5
Exercise Set 1	7
2. A Quick Look at Functions	9
Exercise Set 2	17
3. The Topology of the Real Numbers	21
Exercise Set 3	28
4. Periodic Points and Stable Sets	31
4.1. Graphical Analysis	36
Exercise Set 4	38
5. Sarkovskii's Theorem	41
Exercise Set 5	45
6. Differentiability and Its Implications	47
Exercise Set 6	54

7. Parametrized Families of Functions and Bifurcations	59
Exercise Set 7	67
8. The Logistic Function Part I: Cantor Sets and Chaos	69
8.1. A First Look at the Logistic Function when $r > 4$	70
8.2. Cantor Sets	73
8.3. Chaos and the Dynamics of the Logistic Function	76
8.4. A Few Additional Comments on Cantor Sets	84
Exercise Set 8	84
9. The Logistic Function Part II: Topological Conjugacy	87
Exercise Set 9	92
10. The Logistic Function Part III: A Period-Doubling Cascade	95
Exercise Set 10	104
11. The Logistic Function Part IV: Symbolic Dynamics	109
11.1. Symbolic Dynamics and Metric Spaces	109
11.2. Symbolic Dynamics and the Logistic Function	118
Exercise Set 11	122
12. Newton's Method	127
12.1. Newton's Method for Quadratic Functions	133
12.2. Newton's Method for Cubic Functions	138
12.3. Intervals and Rates of Convergence	145
Exercise Set 12	147
13. Numerical Solutions of Differential Equations	153
Exercise Set 13	163
14. The Dynamics of Complex Functions	167
14.1. The Complex Numbers	167
14.2. Complex Functions	170
14.3. The Dynamics of Complex Functions	174
14.4. The Riemann Sphere	178
14.5. Newton's Method in the Complex Plane	182
Exercise Set 14	188
15. The Quadratic Family and the Mandelbrot Set	193
15.1. Generating Julia and Mandelbrot Sets on a Computer	199
Exercise Set 15	200
Appendix. Mathematica Algorithms	203
A.1. Iterating Functions	204
Finding the Value of a Point Under Iteration	204
Tables of Iterates	204

Controlling the Precision of the Computations	205
Graphing Iterated Functions	206
A.2. Graphical Analysis	206
A.3. Bifurcation Diagrams	208
A.4. Julia Sets	209
A.5. The Mandelbrot Set	211
A.6. Stable Sets of Newton's Method	212
References	215
Dynamical Systems	215
General Interest Books on Dynamics	217
Topics in Mathematics	218
Computer Programs and Algorithms	219
Index	221