

P.J. Hilton U. Stammbach

A Course in Homological Algebra

Second Edition



Springer

Peter J. Hilton
Department of Mathematical Sciences
State University of New York
Binghamton, NY 13902-6000
USA

and

Department of Mathematics
University of Central Florida
Orlando, FL 32816
USA

Urs Stambach
Mathematik
ETH-Zentrum
CH-8092 Zürich
Switzerland

Editorial Board

S. Axler
Department of
Mathematics
Michigan State University
East Lansing, MI 48824
USA

F.W. Gehring
Department of
Mathematics
University of Michigan
Ann Arbor, MI 48109
USA

P.R. Halmos
Department of
Mathematics
Santa Clara University
Santa Clara, CA 95053
USA

Mathematics Subject Classification (1991): 18Axx, 18Gxx, 13Dxx, 16Exx, 55Uxx

Library of Congress Cataloging-in-Publication Data
Hilton, Peter John.

A course in homological algebra / P.J. Hilton, U. Stambach. —
2nd ed.

p. cm. — (Graduate texts in mathematics ; 4)

Includes bibliographical references and index.

ISBN 978-1-4612-6438-5 ISBN 978-1-4419-8566-8 (eBook)

DOI 10.1007/978-1-4419-8566-8

1. Algebra, Homological. I. Stambach, Urs. II. Title.

III. Series.

QA169.H55 1996

512'.55—dc20

96-24224

Printed on acid-free paper.

© 1997 Springer Science+Business Media New York

Originally published by Springer-Verlag New York, Inc. in 1997

Softcover reprint of the hardcover 2nd edition 1997

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag New York, Inc., 175 Fifth Avenue, New York, NY 10010, USA), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production managed by Francine McNeill; manufacturing supervised by Jacqui Ashri.

Typeset by Asco Trade Typesetting Ltd., Hong Kong.

9 8 7 6 5 4 3 2 1

ISBN 978-1-4612-6438-5

SPIN 10541600

Contents

Preface to the Second Edition	vii
Introduction	1
I. Modules	10
1. Modules	11
2. The Group of Homomorphisms	16
3. Sums and Products	18
4. Free and Projective Modules	22
5. Projective Modules over a Principal Ideal Domain	26
6. Dualization, Injective Modules	28
7. Injective Modules over a Principal Ideal Domain	31
8. Cofree Modules	34
9. Essential Extensions	36
II. Categories and Functors	40
1. Categories	40
2. Functors	44
3. Duality	48
4. Natural Transformations	50
5. Products and Coproducts; Universal Constructions	54
6. Universal Constructions (Continued); Pull-backs and Push-outs	59
7. Adjoint Functors	63
8. Adjoint Functors and Universal Constructions	69
9. Abelian Categories	74
10. Projective, Injective, and Free Objects	81
III. Extensions of Modules	84
1. Extensions	84
2. The Functor Ext	89
3. Ext Using Injectives	94
4. Computation of some Ext -Groups	97

5. Two Exact Sequences	99
6. A Theorem of Stein-Serre for Abelian Groups	106
7. The Tensor Product	109
8. The Functor Tor	112
IV. Derived Functors	116
1. Complexes	117
2. The Long Exact (Co)Homology Sequence.	121
3. Homotopy	124
4. Resolutions.	126
5. Derived Functors	130
6. The Two Long Exact Sequences of Derived Functors	136
7. The Functors Ext_A^n Using Projectives	139
8. The Functors $\overline{\text{Ext}}_A^n$ Using Injectives	143
9. Ext^n and n -Extensions	148
10. Another Characterization of Derived Functors	156
11. The Functor Tor_n^A	160
12. Change of Rings	162
V. The Künneth Formula	166
1. Double Complexes	167
2. The Künneth Theorem	172
3. The Dual Künneth Theorem	177
4. Applications of the Künneth Formulas	180
VI. Cohomology of Groups.	184
1. The Group Ring	186
2. Definition of (Co)Homology	188
3. H^0, H_0	191
4. H^1, H_1 with Trivial Coefficient Modules	192
5. The Augmentation Ideal, Derivations, and the Semi-Direct Product	194
6. A Short Exact Sequence	197
7. The (Co)Homology of Finite Cyclic Groups.	200
8. The 5-Term Exact Sequences	202
9. H_2 , Hopf's Formula, and the Lower Central Series	204
10. H^2 and Extensions	206
11. Relative Projectives and Relative Injectives	210
12. Reduction Theorems.	213
13. Resolutions	214
14. The (Co)Homology of a Coproduct	219

15. The Universal Coefficient Theorem and the (Co)Homology of a Product	221
16. Groups and Subgroups	223
VII. Cohomology of Lie Algebras	229
1. Lie Algebras and their Universal Enveloping Algebra	229
2. Definition of Cohomology; H^0, H^1	234
3. H^2 and Extensions	237
4. A Resolution of the Ground Field K	239
5. Semi-simple Lie Algebras	244
6. The two Whitehead Lemmas	247
7. Appendix: Hilbert's Chain-of-Syzygies Theorem	251
VIII. Exact Couples and Spectral Sequences	255
1. Exact Couples and Spectral Sequences	256
2. Filtered Differential Objects	261
3. Finite Convergence Conditions for Filtered Chain Complexes	265
4. The Ladder of an Exact Couple	269
5. Limits	276
6. Rees Systems and Filtered Complexes	281
7. The Limit of a Rees System	288
8. Completions of Filtrations	291
9. The Grothendieck Spectral Sequence	297
IX. Satellites and Homology	306
1. Projective Classes of Epimorphisms	307
2. $\mathcal{E}$ -Derived Functors	309
3. $\mathcal{E}$ -Satellites	312
4. The Adjoint Theorem and Examples	318
5. Kan Extensions and Homology	320
6. Applications: Homology of Small Categories, Spectral Sequences	327
X. Some Applications and Recent Developments	331
1. Homological Algebra and Algebraic Topology	331
2. Nilpotent Groups	335
3. Finiteness Conditions on Groups	339
4. Modular Representation Theory	344
5. Stable and Derived Categories	349

Bibliography 357

Index 359