

Mathematical World

Volume 23

A Mathematical Gift, III

The interplay between
topology, functions,
geometry, and algebra

Koji Shiga
Toshikazu Sunada

Translated by Eiko Tyler

Contents

Preface	vii
Chapter 1. The Story of the Birth of Manifolds	1
Lecture 1. The Prelude to the Birth of Manifolds	3
1.1. 20th Century Mathematics Gave "Birth" to Manifolds	3
1.2. Concepts of a Function	5
1.3. Three Fundamental Properties of Functions—Continuity, Differentiability, and Analyticity	6
1.4. Continuity and Differentiability of Functions of Several Variables	10
1.5. Continuity, Differentiability, and Analyticity in the Development of Mathematics	15
1.6. Arrival of the Notion of a Set	20
1.7. Abstraction of "Nearness"	21
1.8. Coordinate Systems	24
1.9. Coordinate Transformations	26
1.10. A Smooth Place—a Manifold	31
Lecture 2. The Birth of Manifolds	35
2.1. Concept of a Set	35
2.2. Open Sets in \mathbb{R}^n	36
2.3. Three Properties	38
2.4. Definition and Examples of Manifolds	43
2.5. Whitney's Viewpoint	51
2.6. After the Whitney Embedding Theorem	56
Chapter 2. The Story of Area and Volume from Everyday Notions to Mathematical Concepts	61
Lecture 1. Transition from the Notion of "Size" to the Concept of "Area"	63
1.1. "Size" and "Area"	63
1.2. "Area"	68
1.3. Area Function	71
1.4. Uniqueness of the Area Function	73

1.5. Existence of an Area Function	77
Lecture 2. Scissors-Congruent Polygons	89
2.1. Area and Scissors-Congruence	89
2.2. From the History of Mathematics	93
2.3. Square Decomposition of a Rectangle	98
2.4. Infinite Square Decompositions and Continued Fraction Expansion	102
Lecture 3. Scissors-Congruent Polyhedra	109
3.1. Scissors-Congruence of Polyhedra and Dehn's Theorem	109
3.2. Additive Functions	112
3.3. Invariants	114
3.4. Construction of Invariants (Hadwiger's Theorem)	117
3.5. Proof of Dehn's Theorem	122
3.6. Banach-Tarski Paradox—the Mystery of Volume	124
3.7. Complicated Figures	126