

## **The behavior of structures composed of composite materials**

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*By*

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This book is dedicated to our wives Midge and Nina for their patience, encouragement and inspiration.

For Midge Vinson, appreciation is expressed for providing the love, care, and environment for keeping her husband productive and well. This text could not have been completed without her.

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## Preface

While currently available texts dealing with the subject of high performance composite materials touch upon a spectra of topics such as mechanical metallurgy, physical metallurgy, micromechanics and macro-mechanics of such systems, it is the specific purpose of this text to examine elements of the mechanics of structural components composed of composite materials. This text is intended for use in training engineers in this new technology and rational thought processes necessary to develop a better understanding of the behavior of such material systems for use as structural components. The concepts are further exploited in terms of the structural format and development to which the book is dedicated. To this end the development progresses systematically by first introducing the notion and concepts of what these new material classes are, the fabrication processes involved and their unique features relative to conventional monolithic materials. Such introductory remarks, while far too short in texts of this type, appear necessary as a precursor for engineers to develop a better understanding for design purposes of both the threshold limits to which the properties of such systems can be pushed as well as the practical limitations on their manufacture.

Following these introductory remarks, an in-depth discussion of the important differences between composites and conventional monolithic material types is discussed in terms of developing the concepts associated with directional material properties. That is, the ideas of anisotropic elasticity for initially homogeneous bodies in the phenomenological sense are described and presented. The use of such analytical tools is then presented through exemplification of selected problems for a number of classical type problems of various geometric shapes including plane stress, plane strain and the bending of a simply supported beam.

These ideas are carried forward and developed for continuous fiber composites in Chapter Two which discusses both single ply laminae and multi-ply laminate theory. This is then followed by a series of chapters, each of which deals with a functional aspect of structural design in which the basic building blocks of a structural system are made. That is, plates and panels; beams, columns and rods; and cylindrical and spherical shells are each discussed within the framework of their potential use in a

functional environment. Thus the traditional topics of conventional monolithic (isotropic) material structural elements such as structures subjected to static loads, thermal and other environmental loads, structural instability and vibratory response are included along with chapters on energy methods and failure theories of composite materials.

Energy methods have been included to present a tool for solving difficult problems of various types encountered in practice. Indeed, in many instances closed form solutions are not possible and approximate solutions must be sought. Energy methods thus provide both an alternative for the formulation of such problems plus a means of generating approximate solutions.

The chapter on failure theories is a generic presentation in the sense that any and/or all of the above structural components consisting of various multi-ply construction can fail when subjected to a sufficiently large loading combination. It is emphasized that the failure of composites is a complicated, changing issue because of the diverse ways in which such structural systems can fail due both to the geometric ply arrangement of the components, complicated load paths, and the diversity of failure mechanisms which can be activated. Therefore, this chapter should serve in a global sense at best as a guide to the prediction of structural integrity, while more common and acceptable phenomenological failure theories are being developed.

Finally, a chapter on joining is included to discuss to some detail the two methods by which composite material structural components can be joined: namely, adhesive bonding and mechanical fastening. Again, the material presented is an introduction to the subject which is rapidly changing and developing.

At the end of each chapter are several problems, characteristic of the material covered which can be used. Some answers are given in an appendix.

Knowing that nothing is perfect, the authors welcome any notification of errors and ambiguities, and if addresses are provided, authors will forward errata sheets periodically.

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Jack R. Vinson  
Robert L. Sierakowski

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