

Computational Methods for Fluid Dynamics

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Computational Methods for Fluid Dynamics

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Preface

Computational fluid dynamics, commonly known by the acronym ‘CFD’, continues to have significant expansion. There are many software packages available that solve fluid flow problems; thousands of engineers are using them across a broad range of industries and research areas. The market is growing apparently at a rate of around 15% each year. CFD codes are accepted nowadays as design tools in many industries and are used not only to solve problems but also to help in designing and optimizing various products and as a vehicle for research. While the user-friendliness of commercial CFD-tools has greatly increased since the first edition of this book appeared in 1996, for their efficient and reliable application it is still necessary that the user has a solid background in both fluid mechanics and CFD-methods. We assume that our readers are familiar with theoretical fluid mechanics, so we try to provide useful information on the other component—computational methods for fluid dynamics.

The book is based on material offered by the authors in the past in courses at Stanford University, the University of Erlangen-Nürnberg and the Technical University of Hamburg-Harburg, as well as in a number of short courses. It reflects the authors’ experience in developing numerical methods, writing CFD codes and using them to solve engineering and geophysical problems. Many of the codes used in the examples, from the simple ones involving rectangular grids to the ones using non-orthogonal grids and multigrid methods, are available to interested readers; the information on how to access them via the Internet is given in the appendix. These codes illustrate some of the methods described in the book; they can be extended and adapted to the solution of many fluid mechanical problems. Students should try to modify them (e.g., to implement different boundary conditions, interpolation schemes, differentiation and integration approximations, etc.). This is important as one does not really know a method until she or he has programmed and/or run it. We have learned that many researchers have used these codes in the past as the basis for their research projects.

The finite volume method is favored in this book, although finite difference methods are described in what we hope is sufficient detail. Finite element methods are not covered in detail as a number of books on that subject already exist.

The basic ideas of each topic are described in such a way that they can be understood by the reader; where possible, we have avoided lengthy mathematical analysis. Usually, a general description of an idea or method is followed by a more detailed description (including the necessary equations) of one or two numerical schemes representative of the better methods of the type; other possible approaches and extensions are briefly described. We have tried to emphasize common elements of methods rather than their differences and to provide the basis upon which variants can be built.

We have placed considerable emphasis on the need to estimate numerical errors; almost all examples in this book are accompanied with error analysis. Although it is possible for a *qualitatively incorrect* solution of a problem to look reasonable (it may even be a good solution of another problem), the consequences of accepting it may be severe. On the other hand, sometimes a solution of a relatively low accuracy can be of value if treated with care. Industrial users of commercial codes need to learn to judge the quality of the results before believing them. Likewise, researchers have the same challenge. We hope that this book will contribute to the awareness that numerical solutions are always approximate and need to be properly assessed.

We have tried to cover a cross-section of modern approaches, including arbitrary polyhedral and overlapping grids, multigrid methods and parallel computing, methods for moving grids and free surface flows, direct and large-eddy simulation of turbulence, etc. Obviously, we could not cover all these topics in detail, but we hope that the information contained herein will provide the reader with a useful general knowledge of the subject; those interested in a more detailed study of a particular topic will find recommendations for further reading.

The long time between the previous and the current edition of this book was caused by the sudden passing of Joel H. Ferziger in 2004. Although the remaining co-author of the previous edition found an excellent partner for continuing the project in Bob Street, for various reasons (but mostly the lack of time), it took a while before this new edition was finished. The new co-author has brought a new expertise, and the time passed also required that most chapters be significantly revised. Notably, the former Chap. 7 dealing with methods for solving the Navier-Stokes equations has been completely re-written and broken-up into two chapters. Fractional-step methods, which are widely used for large-eddy simulations, have been described in more detail and a new, implicit version has been derived. New codes based on the fractional-step method have been added to the set which can be downloaded by readers from the web-site created specially for that purpose (www.cfd-peric.de). Most examples in later chapters have been re-computed using commercial software; the simulation files for these examples with some instructions can also be downloaded from the above web-site.

While we have invested every effort to avoid typing, spelling, and other errors, no doubt some remain to be found by readers. We will appreciate your notifying us of any mistakes you might find, as well as your comments and suggestions for improvement of future editions of the book. For that purpose, the authors' electronic mail addresses are given below. Corrections, as well as additional extended reports on some of the examples will become available for download at the above web-site.

We also hope that colleagues whose work has not been referenced will forgive us, because any omissions are unintentional.

We have to thank all our present and former students, colleagues, and friends, who helped us in one way or another to finish this work; the complete list of names is too long to present here. Names that we cannot avoid mentioning include (in alphabetic order) Drs. Steven Armfield, David Briggs, Fotini (Tina) Katapodes Chow, Ismet Demirdžić, Gene Golub, Sylvain Lardeau, Željko Lilek, Samir Muzaferija, Joseph Oliger, Eberhard Schreck, Volker Seidl, and Kishan Shah. The help provided by those people who created and made available \TeX , \LaTeX , Linux, Xfig, Gnuplot, and other tools which made our job easier is also greatly appreciated. Special thanks to Rafael Ritterbusch who provided the fluid-structure interaction example in Chap. 13.

Our families gave us a tremendous support during this endeavor; our special thanks go to Eva Ferziger, Anna James, Robinson and Kerstin Perić and Norma Street.

The initial collaboration between geographically distant colleagues was made possible by grants and fellowships from the Alexander von Humboldt Foundation (to JHF) and the Deutsche Forschungsgemeinschaft (German National Research Foundation, to MP). Without their support, this work would never have come into existence and we cannot express sufficient thanks to them. One of the authors (MP) is especially indebted to the late Peter S. MacDonald, former president of CD-adapco, for his support, and to managers at Siemens (Jean-Claude Ercollanely, Deryl Sneider, and Sven Enger), who provided both support and the software Simcenter STAR-CCM+ to create examples in Chaps. 9–13¹. RLS is deeply appreciative of the opportunity to contribute to the continuation of the work of his great friend and research colleague, Joel Ferziger.

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¹Examples in Sects. 9.12.2, 10.3.5, 10.3.8, 12.2.2, 12.5.2, 12.6.4, and all of Chap. 13 (except where another source is explicitly named) were simulated and the images were created with Simcenter STAR-CCM+, a trademark or registered trademark of Siemens Industry Software NV and any of its affiliates.

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Acronyms

1D	One-dimensional
2D	Two-dimensional
3D	Three-dimensional
ADI	Alternating direction implicit
ALM	Actuator line model
BDS	Backward difference scheme
CDS	Central difference scheme
CFD	Computational fluid dynamics
CG	Conjugate gradient method
CGSTAB	CG stabilized
CM	Control mass
CV	Control volume
CVFEM	Control-volume-based finite element method
DDES	Delayed detached-eddy simulation
DES	Detached-eddy simulation
DNS	Direct numerical simulation
EARSM	Explicit algebraic Reynolds-stress model
EB	Elliptic blending
ENO	Essentially non-oscillatory
FAS	Full approximation scheme
FD	Finite difference
FDS	Forward difference scheme
FE	Finite elements
FFT	Fast Fourier transform
FMG	Full multigrid method
FV	Finite volume
GC	Global communication
GS	Gauss-Seidel method
ICCG	CG preconditioned by incomplete Cholesky method
IDDES	Improved delayed detached-eddy simulation

IFSM	Implicit fractional-step method
ILES	Implicit large-eddy simulation
ILU	Incomplete lower-upper decomposition
LC	Local communication
LES	Large-eddy simulation
LU	Lower-upper decomposition
MAC	Marker-and-cell
MG	Multigrid
MPI	Message-passing interface
ODE	Ordinary differential equation
PDE	Partial differential equation
PVM	Parallel virtual machine
RANS	Reynolds averaged Navier-Stokes
rms	Root mean square
rpm	Revolutions per minute
RSFS	Resolved sub-filter scale
RSM	Reynolds-stress model
SBL	Stable boundary layer
SCL	Space conservation law
SFS	Sub-filter scale
SGS	Subgrid scale
SIP	Strongly implicit procedure
SOR	Successive over-relaxation
SST	Shear stress transport
TDMA	Tridiagonal matrix algorithm
TRANS	Transient RANS
TVD	Total variation diminishing
UDS	Upwind difference scheme
URANS	Unsteady RANS
VLES	Very-large-eddy simulation
VOF	Volume-of-fluid