

Sándor Szabó

# **Topics in Factorization of Abelian Groups**

Birkhäuser Verlag  
Basel · Boston · Berlin

# Contents

Preface	vii
<b>1 Hajós's and Rédei's theorems</b>	<b>1</b>
1.1 Introduction . . . . .	1
1.2 Simulated factorizations . . . . .	7
1.3 Hajós's theorem . . . . .	13
1.4 Rédei's theorem . . . . .	20
1.5 Keller's conjecture . . . . .	26
<b>2 Elementary arguments</b>	<b>33</b>
2.1 An axiomatic approach . . . . .	33
2.2 Direct product . . . . .	39
2.3 Nonperiodic factorizations . . . . .	47
<b>3 The machinery</b>	<b>57</b>
3.1 Vanishing sums . . . . .	57
3.2 Annihilators . . . . .	66
3.3 Zero divisors . . . . .	78
<b>4 Four characterization results</b>	<b>89</b>
4.1 Nonsubgroup factors . . . . .	89
4.2 Multiple factorizations . . . . .	93
4.3 Multiple cyclic factorizations . . . . .	99
4.4 Robinson's result . . . . .	108
<b>5 Applying the machinery</b>	<b>123</b>
5.1 Size of annihilators . . . . .	123
5.2 Cyclic type subsets . . . . .	131
5.3 Simulated subsets . . . . .	138
5.4 Periodic subsets . . . . .	146

<b>6 Cyclic prime component</b>	<b>153</b>
6.1 Cyclic prime component . . . . .	153
6.2 Cyclic groups . . . . .	157
6.3 Nonperiodic factorizations . . . . .	161
6.4 Simulated subsets . . . . .	167
<b>7 <math>p</math>-groups</b>	<b>175</b>
7.1 Groups of order $p^4$ . . . . .	175
7.2 Factoring 2-groups . . . . .	182
7.3 Nonperiodic factorizations . . . . .	195
7.4 Elementary 2-component . . . . .	201
<b>8 The Hajós property</b>	<b>215</b>
8.1 Groups without Hajós property . . . . .	215
8.2 Groups with Hajós property . . . . .	221
<b>9 The Rédei property</b>	<b>233</b>
9.1 Latin squares . . . . .	233
9.2 The Rédei property . . . . .	239
9.3 Groups with Rédei property . . . . .	252
<b>10 Infinite groups</b>	<b>263</b>
10.1 Infinite constructions . . . . .	263
10.2 Infinite abelian groups . . . . .	269
<b>11 Further topics</b>	<b>279</b>
11.1 Some applications . . . . .	279
11.2 Miscellaneous results . . . . .	287
<b>A Simulated subsets</b>	<b>297</b>
<b>B Hajós's theorem</b>	<b>301</b>
<b>C Rédei's theorem</b>	<b>307</b>
<b>D Background of constructions</b>	<b>311</b>
<b>E Cyclotomic polynomials</b>	<b>317</b>
<b>References</b>	<b>321</b>
<b>Index</b>	<b>332</b>