Robert S. Liptser  Albert N. Shiryaev

Statistics of Random Processes

II. Applications

Translated by A. B. Aries
Translation Editor: Stephen S. Wilson

Second, Revised and Expanded Edition
# Table of Contents

Preface to the Second Edition ........................................... V

11. Conditionally Gaussian Processes ................................. 1
   11.1 Assumptions and Formulation of the Theorem of Conditional
        Gaussian Behavior ............................................ 1
   11.2 Auxiliary Lemmas ............................................. 3
   11.3 Proof of the Theorem of Conditional Gaussian Behavior .... 9

12. Optimal Nonlinear Filtering: Interpolation and Extrapolation
    of Components of Conditionally Gaussian Processes ......... 17
   12.1 Optimal Filtering Equations .................................. 17
   12.2 Uniqueness of Solutions of Filtering Equations: Equivalence
        of $\sigma$-Algebras $F_t^\xi$ and $F_t^{\xi_0,\bar{W}}$ ................. 25
   12.3 Optimal Filtering Equations in Several Dimensions .......... 32
   12.4 Interpolation of Conditionally Gaussian Processes .......... 38
   12.5 Optimal Extrapolation Equations ............................. 49

13. Conditionally Gaussian Sequences: Filtering and Related
    Problems .......................................................... 55
   13.1 Theorem on Normal Correlation ................................ 55
   13.2 Recursive Filtering Equations for Conditionally
        Gaussian Sequences ........................................... 67
   13.3 Forward and Backward Interpolation Equations .............. 77
   13.4 Recursive Equations of Optimal Extrapolation ............. 88
   13.5 Examples ...................................................... 91

    of Random Sequences ............................................. 99
   14.1 Optimal Linear Filtering of Stationary Sequences
        with Rational Spectra ......................................... 99
   14.2 Maximum Likelihood Estimates for Coefficients
        of Linear Regression ......................................... 107
   14.3 A Control Problem with Incomplete Data (Linear System
        with Quadratic Performance Index) ......................... 113
# Table of Contents

14.4 Asymptotic Properties of the Optimal Linear Filter ........... 121  
14.5 Recursive Computation of the Best Approximate Solutions  
(Pseudo-solutions) of Linear Algebraic Systems ............... 132  
14.6 Kalman Filter under Wrong Initial Conditions ............. 138

15. Linear Estimation of Random Processes .................... 145  
15.1 Wide-Sense Wiener Processes ................................ 145  
15.2 Optimal Linear Filtering for some Classes of Nonstationary  
Processes .................................................. 157  
15.3 Linear Estimation of Wide-Sense Stationary Random Pro-  
cesses with Rational Spectra .............................. 161  
15.4 Comparison of Optimal Linear and Nonlinear Estimates .... 170

16. Application of Optimal Nonlinear Filtering Equations to  
some Problems in Control Theory and Estimation Theory 177  
16.1 An Optimal Control Problem Using Incomplete Data ...... 177  
16.2 Asymptotic Properties of Kalman–Bucy Filters .......... 184  
16.3 Computation of Mutual Information and Channel Capacity  
of a Gaussian Channel with Feedback ....................... 190  
16.4 Optimal Coding and Decoding for Transmission of a Gaussian  
Signal Through a Channel with Noiseless Feedback .......... 195  
16.5 Asymptotic Properties of the Linear Filter under Wrong  
Initial Conditions ......................................... 214

17. Parameter Estimation and Testing of Statistical Hypotheses  
for Diffusion-Type Processes ................................ 219  
17.1 Maximum Likelihood Method for Coefficients  
of Linear Regression ....................................... 219  
17.2 Parameter Estimation of the Drift Coefficient  
for Diffusion-Type Processes ............................... 225  
17.3 Parameter Estimation of the Drift Coefficient  
for a One-Dimensional Gaussian Process ................. 230  
17.4 Two-Dimensional Gaussian Markov Processes:  
Parameter Estimation ....................................... 236  
17.5 Sequential Maximum Likelihood Estimates ................ 244  
17.6 Sequential Testing of Two Simple Hypotheses  
for Itô Processes ........................................... 248  
17.7 Some Applications to Stochastic Approximation .......... 256

18. Random Point Processes: Stieltjes Stochastic Integrals .... 261  
18.1 Point Processes and their Compensators .................. 261  
18.2 Minimal Representation of a Point Process: Processes of the  
Poisson Type ................................................. 269  
18.3 Construction of Point Processes with Given Compensators:  
Theorems on Existence and Uniqueness ........................ 277
18.4 Stieltjes Stochastic Integrals .................................. 286
18.5 The Structure of Point Processes with Deterministic
and Continuous Compensators .................................. 305

19. The Structure of Local Martingales, Absolute Continuity
of Measures for Point Processes, and Filtering ............ 309
19.1 The Structure of Local Martingales ....................... 309
19.2 Nonnegative Supermartingale: Analog of Girsanov’s Theorem 315
19.3 Optimal Filtering from the Observations of Point Processes .. 325
19.4 The Necessary and Sufficient Conditions for Absolute Continuity of the Measures Corresponding to Point Processes .... 336
19.5 Calculation of the Mutual Information and the Cramer–Rao–Wolfowitz Inequality (the Point Observations) ............... 345

20. Asymptotically Optimal Filtering ......................... 355
20.1 Total Variation Norm Convergence and Filtering .......... 355
20.2 Robust Diffusion Approximation for Filtering ............ 371

Bibliography .......................................................... 383

Index ................................................................. 399